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E_{11}, ten forms and supergravity

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Abstract: We extend the previously given non-linear realisation of E_{11} for the decomposition appropriate to IIB supergravity to include the ten forms that were known to be present in the adjoint representation. We find precise agreement with the results on ten forms found by closing the IIB supersymmetry algebra.

Keywords: M-Theory, Supergravity Models, Space-Time Symmetries.

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Long ago it was realised that the maximal supergravity theories in ten and eleven dimensions [1]–[4] when dimensionally reduced on a torus lead to maximal supergravity theories which possessed unexpected symmetries. In particular, the eleven dimensional supergravity theory when dimensionally reduced on a torus of dimension n possess an E_n symmetry for $n \leq 8$ [5], with some evidence [6] for an E_9 symmetry when reduced to two dimensions and it has been conjectured [7] to have a E_{10} symmetry in one dimension. The scalar fields which are created by the dimensional reduction process belong to a coset, or non-linear realisation, based on an E_n algebra with the local sub-algebra being the Cartan involution invariant sub-algebra.

In more recent years, it was realised [8] that the entire bosonic sector of of eleven dimensional supergravity, including gravity, could be formulated as a non-linear realisation [4]. In this construction, the presence of gravity requires an A_{10} algebra together with other generators, which transform as tensors under this A_{10} algebra, and have non-trivial commutation relations amongst themselves that are determined by the dynamics of the theory. When formulated in this way it becomes apparent that the eleven dimensional supergravity theory may be part of a larger theory, and assuming that this is a non-linearly realised Kac-Moody algebra, one finds that it must contain a rank eleven algebra called E_{11} [9]. A similar chain of argument applies to the bosonic sectors of the IIA and IIB supergravity theories which are also thought to be part of larger theories that are non-linear realisations of E_{11} [9, 10].

Similar ideas were subsequently taken up by the authors of reference [14] who considered the idea that the eleven dimensional supergravity theory is a non-linear realisation of the E_{10} sub-algebra of E_{11}. However, these authors proposed that space-time was in fact contained within E_{10}. A hybrid proposal based on E_{11}, but adopting similar ideas to the latter for space-time was also given [15].

We invite the reader to draw the Dynkin diagram of E_{11} by drawing ten nodes connected together by a single horizontal line. We label these nodes from left to right by the integers from one to ten and then add a further node, labeled eleven, above node eight and attached by a single vertical line. The latter node is sometimes called the exceptional node. We refer the reader to earlier works of the author for a brief review of Kac-Moody algebras useful for the considerations of this paper.

The eleven dimensional, IIA and IIB theories are thought to all have an underlying E_{11} symmetry which is non-linearly realised with the local sub-algebra being the Cartan involution invariant sub-algebra. As a result, in the non-linear realisation the group element contains positive root and Cartan sub-algebra generators whose coefficients turn out to be the fields of the theory. The gravity sector is associated with a A_{D-1} type sub-algebra, where D is the space-time dimension of the theory, which arises as a sub-Dynkin diagram that contains node one and a set of continuously connected nodes of the E_{11} Dynkin diagram. This set of nodes is referred to as the gravity line. The eleven dimensional, IIA and IIB theories are distinguished by their different gravity sub-algebras, or gravity lines. The eleven dimensional theory must possess an A_{10} gravity algebra and there is only one such algebra whose gravity line contains all the nodes except node eleven. For this theory it is useful to classify the E_{11} algebra in terms of generators that transform under this A_{10} sub-algebra.

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The IIA and IIB theories are ten dimensional and to find these theories we seek an A_9 gravity sub-algebra and so we must choose the gravity line to be a sub-Dynkin diagram that consist of nine nodes. Looking at the E_{11} Dynkin diagram there are only two ways to do this. Starting from the node labeled one we must choose a A_9 sub-Dynkin diagram, but once we get to the junction of the E_{11} Dynkin diagram, situated at the node labeled 8, we can continue along the horizontal line with two further nodes taking only the first node to belong to the A_9, or we can find the final A_9 node by taking it to be the only node in the other choice of direction at the junction. These two ways correspond to the IIA and IIB theories respectively. Hence, in the IIA theory we take the gravity line to be nodes labeled one to nine inclusive while for the IIB theory the gravity line contains nodes one to eight and in addition node eleven [9, 10]. For these two theories it is useful to classify the E_{11} algebra in terms of their respective A_9 sub-algebras, but as these are different embeddings in E_{11} we find different field contents.

While the number and type of generators is not known for any Kac-Moody algebra one can find them at low levels. Every generator corresponds to a root in the Kac-Moody algebra which can be written in terms of an integer sum of the simple roots. By definition a Lorentzian Kac-Moody algebra is one which possess a Dynkin diagram which has one node whose deletion leads to a Dynkin diagram that corresponds to finite algebra together with possibly only one affine algebra [16]. For the E_{11} Dynkin diagram we may delete node eleven to obtain an A_{10} sub-algebra and so E_{11} is a Lorentzian Kac-Moody algebra. The advantage of such algebra is that one can study its properties in terms of the remaining sub-algebra, or algebras, whose representations are well known. In particular we may decompose the Lorentzian algebra, meaning its adjoint representation, into representations of the sub-algebra. The representations of the latter are determined by their highest weights. A given highest weight will appear in a particular root of the Lorentzian algebra and the number of times the roots of the deleted nodes appear in this root are called the levels and can be used to label the representations of the sub-algebra that appear in the decomposition [14, 17]. For example, deleting node eleven in the E_{11} Dynkin diagram we obtain an A_{10} sub-algebra whose decomposition with respect to which is appropriate to the eleven dimensional theory. Carrying this out, one finds at low levels that the algebra contains the generators of A_{10}, and then a three form and six form generator as well as a generator with eight indices anti-symmetrised and a further index. In the non-linear realisation these generators correspond to gravity, the three form gauge field and its dual, and dual graviton respectively, which is the field content of the eleven dimensional supergravity theory [9]. There are, of course, an infinite number of generators, and so fields, at higher levels.

By deleting nodes nine and ten we decompose the E_{11} algebra with respect to an A_9 algebra that is the one appropriate to the IIB theory. The representations are labeled by two integers corresponding to the nodes deleted and are listed in the table on page 27 of reference [12]. As first noticed in reference [10] one finds at low levels a set of generators that correspond precisely to the field content of the IIB supergravity theory and their duals. Indeed, if one includes the dual of gravity, it is very striking how this accounts for the first nine entries of the table. However, there are at higher levels an infinite number of other fields. Among these one finds an additional eight form which, together with the earlier ones form an SU(1,1) triplet. One also finds some ten forms which form a doublet and quadruplet of SU(1,1) [12]. The triplet of eight forms was first observed in reference [18]. Although the ten form fields have no dynamics they couple to space-filling branes which are dynamical. Their existence was first considered as a result of a string world-sheet analysis of D branes considerations [21]. Two ten forms were also observed in the context of IIB supergravity in reference [23], but it was shown in reference [24] that these could not be a doublet of SU(1,1). The eight and ten form fields fields have more subsequently been seen from an entirely different view point. The authors of reference [11] considered what eight and ten forms could be added to the IIB theory such that the supersymmetry algebra still closed. They found precisely the eight and ten forms predicted by E_{11}. These authors also found, using the same calculation, the gauge transformations of all the gauge fields including the eight and ten form fields and constructed some gauge invariant quantities.

In this paper we extend the calculation of reference [10] to include one further eight form and the ten forms and compute the E_{11} invariant Cartan forms constructed from the gauge fields. We find that these are in precise agreement with gauge invariant objects computed using the closure of the supersymmetry algebra in reference [11].

The Kac-Moody algebra E_{10} considered in reference [14] does not possess [20] the ten forms that occur in the E_{11} theory and whose presence has been confirmed in the IIB supergravity theory.

As explained above, the IIB theory emerges from the E_{11} algebra by taking the decomposition with respect to a particular A_9 algebra, hence forth denoted $\hat{A}_9$, whose Dynkin diagram is embedded in that of E_{11} by taking nodes labeled one to eight and node eleven. The latter is the so called exceptional node of the algebra. Carrying out the non-linear realisation one finds that the $\hat{A}_9$ algebra is associated with the gravity fields of the IIB theory and we denote its generators by $\hat{K}^a{}_b$. The nodes not included in the $\hat{A}_9$ sub-algebra, or gravity line, are the nodes labeled nine and ten of the E_{11} Dynkin diagram which are therefore the ones that must be deleted to find the $\hat{A}_9$ decomposition of the E_{11} algebra. The $\hat{A}_9$ representations in the decomposition are then labeled by the levels associated with these two nodes, that is the number of times these two roots occur in the E_{11} root that contains the $\hat{A}_9$ highest weight of the representation under consideration. The decomposition with this labeling is given in table on page 27 in reference [12]. The E_{11} algebra is generated by the Chevalley generators $H_a, E_a, F_a, a = 1, \ldots, 11$. The SU(1,1) invariance of the IIB theory is easy to see from the E_{11} view point as it is just the A_1 algebra associated with node ten. As this is not directly connected to the gravity line of the IIB theory it is an internal symmetry . Thus the SU(1,1) is generated by the H_{10}, E_{10} and F_{10} generators. In fact, one can just delete node nine as then the E_{11} Dynkin diagram splits into two pieces corresponding to $\hat{A}_9$ and A_1 which classify the representations corresponding to the deletion of this node. It is straightforward to collect the generators in the table of reference [12] at a given level corresponding to the root α_9 into multiplets of A_1.

When constructing the E_{11} non-realisation the E_{11} group element contains the Cartan sub-algebra elements and the positive root generators whose coefficients are the fields of the theory. However, the description of the E_{11} algebra from the mathematical view point does not lead to the usual fields that appear in the supergravity theories. The latter, that

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is the physical fields, arise as coefficients of linear combinations of the generators used in the mathematical formulation of the E_{11} algebra. In particular, the Cartan sub-algebra of E_{11} contains the generators $H_a, a = 1, \ldots, 11$ when formulated in terms of its Chevalley basis. Their relation to the generators associated to the fields of the IIB theory, which are given a hat, is given by [10]

$$\begin{aligned} H_a &= \hat{K}^a{}_a - \hat{K}^{a+1}{}_{a+1}, a = 1, \ldots, 8,\ H_9 = \hat{K}^9{}_9 + \hat{K}^{10}{}_{10} + \hat{R} - \frac{1}{4}\sum_{a=1}^{10} \hat{K}^a{}_a, \\ H_{10} &= -2\hat{R}_1,\ H_{11} = \hat{K}^9{}_9 - \hat{K}^{10}{}_{10} \end{aligned} \tag{1}$$

The generator $\hat{R}_1$ will turn out to be associated with the dilaton σ of the IIB theory in the non-linear realisation as traditionally normalised.

The positive root Chevalley generators $E_a, a = 1, \ldots, 11$ of E_{11} are given by [10]

$$E_a = \hat{K}^a{}_{a+1}, a = 1, \ldots 8,\ E_9 = \hat{R}_1^{910},\ E_{10} = \hat{R}_2,\ E_{11} = \hat{K}^9{}_{10}. \tag{2}$$

where the generators $\hat{R}_1^{ab}$ and $\hat{R}_2$ are associated with the NS-NS two form and the axion, $\hat{\chi}$ of the IIB theory respectively. The last equation reflects the fact that the node labeled eleven is the last node in the IIB gravity line, but is the exceptional node of the E_{11} algebra.

The E_{11} algebra is just multiple commutators of the E_a, and separately the F_a generators, subject to the Serre relations. However, it is more efficient to construct the algebra using the list of generators with the $\hat{A}_9$ decomposition given in the table on page 27 of reference [12] and then ensuring that the Jacobi identities are satisfied. This was done for all the generators which in the non-linear realisation are associated with the fields of the IIB supergravity theory and their duals in reference [10] and extended to higher levels in reference [13]. This construction also included the generator corresponding to the the dual of the gravity field, two eight form generators, which are duals of the scalar fields and in addition one of the ten form generators. Examining the table of reference [12], we find that it contains at low levels three eight forms which make up a triplet as well as a doublet and quadruplet of ten forms. We now extend this construction of the algebra to include the third of the eight form generators and all the other ten form generators. It will be advantageous to do this in such a way that the A_1 character of the fields are manifest. Since the part of the theory we wish to test concerns the gauge fields we will not explicitly discuss the generators $\hat{K}^a{}_b$ of the $\hat{A}_9$ and set to zero the generator $R^{a_1 \ldots a_7, b}$, corresponding to the dual of gravity, when it appears on the right hand side of the commutators. Considering IIB table of reference [12], and taking the last comment into account, we introduce the positive root generators of E_{11} not in the Cartan sub-algebra in the form

$$E_{10}, T_\alpha^{a_1 a_2}, T^{a_1 \ldots a_4}, T_\alpha^{a_1 \ldots a_6}, T_{\alpha\beta}^{a_1 \ldots a_8}, T_{\alpha\beta\gamma}^{a_1 \ldots a_{10}}, T_\alpha^{a_1 \ldots a_{10}}, \ldots \tag{3}$$

The E_{11} algebra for these generators is given by

$$\begin{aligned} &[T_\alpha^{a_1 a_2}, T_\beta^{a_3 a_4}] = -\epsilon_{\alpha\beta} T^{a_1 \ldots a_4},\ [T_\alpha^{a_1 a_2}, T^{a_3 \ldots a_6}] = 4 T_\alpha^{a_1 \ldots a_6},\ [T_\alpha^{a_1 a_2}, T_\beta^{a_3 \ldots a_8}] = -T_{\alpha\beta}^{a_1 \ldots a_8} \\ &[T_\alpha^{a_1 a_2}, T_{\beta\gamma}^{a_1 \ldots a_8}] = T_{\alpha\beta\gamma}^{a_1 \ldots a_{10}},\ [T^{a_1 \ldots a_4}, T^{b_1 \ldots b_4}] = 0,\ [T_\alpha^{a_1 \ldots a_6}, T^{b_1 \ldots b_4}] = 0\,. \end{aligned} \tag{4}$$

The SU(1,1) properties of the generators are given by

$$[E_{10}, T^{a_1\ldots a_p}_{1\ldots 112\ldots 2}] = i_1 T^{a_1\ldots a_p}_{1\ldots 122\ldots 2} \tag{5}$$

where i_1 is the number of one indices and the generator on the right-hand side of the commutator has one more 2 index than that in the commutator. We also have

$$[H_{10}, T^{a_1\ldots a_p}_{\alpha_1\ldots\alpha_r}] = (2(\alpha_1 + \ldots + \alpha_r) - 3r) T^{a_1\ldots a_p}_{\alpha_1\ldots\alpha_r}\,. \tag{6}$$

In deriving these equations we have used that E_{10} acts as a lower operator for the representations of SU(1,1), taking $T_1^{a_1a_2} = R_1^{a_1a_2}$, that is $T_1^{910} = E_9$, and normalising $T_2^{a_1a_2}$ such that $[E_{10}, T_1^{a_1a_2}] = T_2^{a_1a_2}$. Then using equations (5) and (6) and the Jacobi identities, and the defining relations $[H_{10}, E_{10}] = 2E_{10}$ and $[H_{10}, E_9] = -E_9$ we find the above equations.

The relation to the generators used in references [10] and [13] is given by

$$\begin{aligned} T_\alpha^{a_1a_2} &= R_\alpha^{a_1a_2},\ T^{a_1\ldots a_4} = R_2^{a_1\ldots a_4},\ T_\alpha^{a_1\ldots a_6} = -\epsilon_{\alpha\beta} R_\beta^{a_1\ldots a_6},\ T_{11}^{a_1\ldots a_8} = R_2^{a_1\ldots a_8}, \\ T_{12}^{a_1\ldots a_8} &= -\frac{1}{2} R_1^{a_1\ldots a_8},\ T_{22}^{a_1\ldots a_8} = -S_2^{a_1\ldots a_8},\ T_{111}^{a_1\ldots a_{10}} = R_2^{a_1\ldots a_{10}} \end{aligned} \tag{7}$$

from which one can verify that, in the absence of one of the eight form generators and three of the ten form generators, the above commutators agree with those of references [10] and [13]. Although it may appear that at first sight the generator $T_\alpha^{a_1\cdots a_{10}}$ can appear on the right hand side of lower level commutators it turns out that this is forbidden by the Jacobi identities if we set to zero the generators associated with the gravity sector. Clearly, as we are dealing with a Kac-Moody, this generator will have to appear as a result of some lower level commutators in the full theory.

The non-linear realisation is by definition a theory which is invariant under $g \to g_0 g h$ where g_0 is a constant E_{11} transformations and H is an element of the local sub-algebra which in this case is the Cartan involution invariant sub-algebra. We may use the latter to gauge away all the negative root terms in the expression for the group element g. As such to construct the non-linear realisation we consider the E_{11} group element given by

$$\begin{aligned} g = {}& e^{\frac{B^\alpha_{a_1\ldots a_{10}}}{10!} T_\alpha^{a_1\ldots a_{10}}} e^{\frac{B^{\alpha\beta\gamma}_{a_1\ldots a_{10}}}{10!} T_{\alpha\beta\gamma}^{a_1\ldots a_{10}}} e^{\frac{B^{\alpha\beta}_{a_1\ldots a_8}}{8!} T_{\alpha\beta}^{a_1\ldots a_8}} e^{\frac{B^\alpha_{a_1\ldots a_6}}{6!} T_\alpha^{a_1\ldots a_6}} \\ & .e^{\frac{B_{a_1\ldots a_4}}{4!} T^{a_1\ldots a_4}} e^{\frac{B^\alpha_{a_1a_2}}{2!} T_\alpha^{a_1a_2}} g_{A_1} \end{aligned} \tag{8}$$

where

$$g_{A_1} = e^{\chi E_{10}} e^{\phi H_{10}}\,. \tag{9}$$

We have, as previously stated, omitted the gravity sector. The Cartan forms are invariant under g_0 transformations and being part of the Lie algebra are of the form

$$\begin{aligned} g^{-1}\partial_\mu g = {}& g_{A_1}^{-1}\Big(\frac{\tilde G^\alpha_{\mu a_1\ldots a_{10}}}{10!} T_\alpha^{a_1\ldots a_{10}} + \frac{\tilde G^{\alpha\beta\gamma}_{\mu a_1\ldots a_{10}}}{10!} T_{\alpha\beta\gamma}^{a_1\ldots a_{10}} + \frac{\tilde G^{\alpha\beta}_{\mu a_1\ldots a_8}}{8!} T_{\alpha\beta}^{a_1\ldots a_8} + \frac{\tilde G^\alpha_{\mu a_1\ldots a_6}}{6!} T_\alpha^{a_1\ldots a_6} \\ & + \frac{\tilde G_{\mu a_1\ldots a_4}}{4!} T^{a_1\ldots a_4} + \frac{\tilde G^\alpha_{\mu a_1a_2}}{2!} T_\alpha^{a_1a_2}\Big) g_{A_1}\,. \end{aligned} \tag{10}$$

Using equation (4) it is straightforward to find that

$$
\begin{aligned}
\tilde{G}^{\alpha}_{\mu a_1 a_2} &= \partial_\mu B^{\alpha}_{a_1 a_2},\ \tilde{G}_{\mu a_1\ldots a_4} = \partial_\mu B_{a_1\ldots a_4} + 3\epsilon_{\alpha\beta} B^{\alpha}_{a_1 a_2}\partial_\mu B^{\beta}_{a_3 a_4},\\
\tilde{G}^{\alpha}_{\mu a_1\ldots a_6} &= \partial_\mu B^{\alpha}_{a_1\ldots a_6} - 6.5.2 B^{\alpha}_{a_1 a_2}(\partial_\mu B_{a_3\ldots a_6} + \epsilon_{\gamma\delta} B^{\gamma}_{a_3 a_4}\partial_\mu B^{\delta}_{a_5 a_6}),\\
\tilde{G}^{\alpha\beta}_{\mu a_1\ldots a_8} &= \partial_\mu B^{\alpha\beta}_{a_1\ldots a_6} +\\
&\quad + 7.4 B^{(\alpha}_{[a_1 a_2}(\partial_\mu B^{\beta)}_{a_3\ldots a_8]} - 6.5 B^{\beta)}_{a_3 a_4}\partial_\mu B_{a_5\ldots a_8]} - 3.5 B^{\beta)}_{a_3 a_4}\epsilon_{\gamma\delta} B^{\gamma}_{a_5 a_6}\partial_\mu B^{\delta}_{a_7 a_8]})\\
\tilde{G}^{\alpha\beta\gamma}_{\mu a_1\ldots a_{10}} &= \partial_\mu B^{\alpha\beta\gamma}_{a_1\ldots a_{10}} - 9.5 B^{(\alpha}_{[a_1 a_2}(\partial_\mu B^{\beta\gamma)}_{a_3\ldots a_{10}} + 2.7 B^{\beta}_{a_3 a_4}\partial_\mu B^{\gamma)}_{a_5\ldots a_{10}]}\\
&\quad -8.7.5 B^{\beta}_{a_3 a_4} B^{\gamma)}_{a_5 a_6}\partial_\mu B_{a_7\ldots a_{10}]} - 7.6.2 B^{\beta}_{a_3 a_4} B^{\gamma)}_{a_5 a_6}\epsilon_{\epsilon\delta} B^{\epsilon}_{a_7 a_8}\partial_\mu B^{\delta}_{a_9 a_{10}]})\\
\tilde{G}^{\alpha}_{a_1\ldots a_{10}} &= \partial_\mu B^{\alpha}_{a_1\ldots a_{10}}\,.
\end{aligned}
\tag{11}
$$

We denote the result of carrying out the evaluation of the final SU(1,1) g_{A_1} factors by

$$
\begin{aligned}
g^{-1}\partial_\mu g = {}& \frac{G^{\alpha}_{\mu a_1\ldots a_{10}}}{10!} T^{a_1\ldots a_{10}}_{\alpha} + \frac{G^{\alpha\beta\gamma}_{\mu a_1\ldots a_{10}}}{10!} T^{a_1\ldots a_{10}}_{\alpha\beta\gamma} + \frac{G^{\alpha\beta}_{\mu a_1\ldots a_8}}{8!} T^{a_1\ldots a_8}_{\alpha\beta} + \frac{G^{\alpha}_{\mu a_1\ldots a_6}}{6!} T^{a_1\ldots a_6}_{\alpha}\\
&+ \frac{G_{\mu a_1\ldots a_4}}{4!} T^{a_1\ldots a_4} + \frac{G^{\alpha}_{\mu a_1 a_2}}{2!} T^{a_1 a_2}_{\alpha} + S^1_\mu \tilde{E}_{10} + S^2_\mu H_{10}\,.
\end{aligned}
\tag{12}
$$

Using equations (5) and (6) one finds that

$$
\begin{aligned}
G^{\alpha}_{\mu a_1 a_2} &= \tilde{G}^{\beta}_{\mu a_1 a_2} U_\beta{}^\alpha,\ G^{\alpha}_{\mu a_1\ldots a_6} = \tilde{G}^{\beta}_{\mu a_1\ldots a_6} U_\beta{}^\alpha,\ G^{\alpha}_{\mu a_1\ldots a_4} = \tilde{G}^{\alpha}_{\mu a_1\ldots a_4} \qquad (13)\\
G^{\alpha\beta}_{\mu a_1\ldots a_8} &= \tilde{G}^{\delta\epsilon}_{\mu a_1\ldots a_6} U_\delta{}^\alpha U_\epsilon{}^\beta,\ G^{\alpha\beta\gamma}_{\mu a_1\ldots a_{10}} = \tilde{G}^{\delta\epsilon\tau}_{\mu a_1\ldots a_{10}} U_\delta{}^\alpha U_\epsilon{}^\beta U_\tau{}^\gamma,\ G^{\alpha}_{\mu a_1\ldots a_6} = \tilde{G}^{\beta}_{\mu a_1\ldots a_6} U_\beta{}^\alpha
\end{aligned}
$$

where

$$
U = \begin{pmatrix} e^{\phi} & -\chi e^{-\phi} \\ 0 & e^{-\phi} \end{pmatrix}\,. \tag{14}
$$

The last two terms in equation (12) are just the standard vierbein and connection on the SU(1,1)/U(1) coset

The Cartan forms are inert under rigid E_{11} transformations, but transform under the local sub-algebra. They do not contain the curl of the gauge fields and so are not invariant under gauge transformations. However, a rigid E_{11} transformation for a particular generator shifts the field corresponding to that generator as well as giving field dependent terms. This transformation can be thought of as a particular gauge transformation. For example under a rigid E_{11} transformation corresponding to the generator $T^{a_1 a_2}_{\alpha}$ we find that $\delta B^{\alpha}_{a_1 a_2} = a^{\alpha}_{a_1 a_2} + \ldots$ where $a^{\alpha}_{a_1 a_2}$ is a constant. This is a gauge transformation $\delta B^{\alpha}_{a_1 a_2} = 2\partial_{[a_1}\lambda^{\alpha}_{a_2]} + \ldots$ with gauge parameter $\Lambda^{\alpha}_{a} = \frac{1}{2} a^{\alpha}_{ab} x^b$. The Cartan forms of equation (11) are used to construct the equations of motion, but to find the field equations of IIB supergravity [10] one used only a sub-set of all the Cartan forms and for the fields with completely anti-symmetrised indices this was the totally anti-symmetrised Cartan forms, that is the field strengths given by

$$
F^{\alpha_1\ldots\alpha_r}_{a_1\ldots a_{p+1}} = (p+1) G^{\alpha_1\ldots\alpha_r}_{[a_1\ldots a_{p+1}]}\,. \tag{15}
$$

The μ index is converted to a tangent index using the delta symbol as we are taking the gravity sector to be trivial. One way to view this enforced anti-symmetrisation is to consider demanding that the theory also be invariant under the simultaneous non-linear realisation of the conformal group. For gravity alone this does pick out particular combinations of the A_{D-1} Cartan forms and one finds it leads uniquely to Einstein's theory [19]. Thus although one started with rigid transformations one ended up with local general coordinate transformations. In fact, the closure of translations and A_{D-1} transformations leads to general coordinate transformations. For gauge fields it is also true that the closure of rigid transformations arising from a non-linear realisation and conformal transformations lead to local symmetries, namely gauge transformations [9]. When the maximal supergravity theories in ten and eleven transformations were found using the E_{11} non-linear realisation it was also combined with the conformal group [9, 10]. Should one not carry out this latter step then one would find the correct equations of motion, but some constant would have to be chosen appropriately. The result of the closure of conformal transformations and E_{11} transformations is unexplored, but it does convert all the E_{11} rigid transformations into local transformations and so the above rigid transformations into gauge transformations. We should note that in finding the equations of motion of the maximal supergravities from the non-linearly E_{11} in references [9, 10] one only required the local Lorentz part of the local sub-algebra and it would be very instructive to enforce the rest of the local sub-algebra up to the level required.

In order to compare the invariant quantities that arise with those in reference [11] we must carry out a field redefinition. In particular, carrying out the field redefinitions

$$\begin{aligned}
C^{\alpha}_{a_1a_2} &= B^{\alpha}_{a_1a_2},\ C_{a_1\ldots a_4} = B_{a_1\ldots a_4},\ C^{\alpha}_{a_1\ldots a_6} = B^{\alpha}_{a_1\ldots a_6} - 5.8B^{\alpha}_{[a_1a_2}B_{a_3\ldots a_6]}\\
C^{\alpha\beta}_{a_1\ldots a_8} &= B^{\alpha\beta}_{a_1\ldots a_8} + 3.7B^{(\alpha}_{[a_1a_2}C^{\beta)}_{a_1\ldots a_6]} + 7.5.4.3B^{(\alpha}_{[a_1a_2}B^{\beta)}_{a_3a_4}B_{a_5\ldots a_8]},\\
C^{\alpha\beta\gamma}_{a_1\ldots a_{10}} &= B^{\alpha\beta\gamma}_{a_1\ldots a_{10}} - 9.4B^{(\alpha}_{[a_1a_2}C^{\beta\gamma)}_{a_3\ldots a_{10}]} + 9.7.5.3B^{(\alpha}_{[a_1a_2}B^{\beta}_{a_3a_4}C^{\gamma)}_{a_5\ldots a_{10}]} +\\
&\quad +16.9.7.5B^{(\alpha}_{[a_1a_2}B^{\beta}_{a_3a_4}B^{\gamma)}_{a_5a_6}B_{a_7\ldots a_{10}]},\ C^{\alpha}_{a_1\ldots a_{10}} = B^{\alpha}_{a_1\ldots a_{10}}
\end{aligned} \tag{16}$$

the Cartan forms become

$$\begin{aligned}
\tilde{G}^{\alpha}_{\mu a_1a_2} &= \partial_\mu C^{\alpha}_{a_1a_2},\ \tilde{G}_{\mu a_1\ldots a_4} = \partial_\mu C_{a_1\ldots a_4} + 3\epsilon_{\alpha\beta}C^{\alpha}_{[a_1a_2}\tilde{G}^{\beta}_{\mu a_3a_4]},\\
\tilde{G}^{\alpha}_{\mu a_1\ldots a_6} &= \partial_\mu C^{\alpha}_{a_1\ldots a_6} - 5.4C^{\alpha}_{[a_1a_2}\tilde{G}_{\mu a_3\ldots a_6]} + 8.5\tilde{G}^{\alpha}_{\mu[a_1a_2}C_{a_3\ldots a_6]},\\
\tilde{G}^{\alpha\beta}_{\mu a_1\ldots a_8} &= \partial_\mu C^{\alpha\beta}_{a_1\ldots a_6} + 7B^{(\alpha}_{[a_1a_2}\tilde{G}^{\beta)}_{\mu a_3\ldots a_8]} - 7.3\tilde{G}^{(\alpha}_{\mu[a_1a_2}C^{\beta)}_{a_3\ldots a_8]}\\
\tilde{G}^{\alpha\beta\gamma}_{\mu a_1\ldots a_{10}} &= \partial_\mu C^{\alpha\beta\gamma}_{a_1\ldots a_{10}} - 9C^{(\alpha}_{[a_1a_2}\tilde{G}^{\beta\gamma)}_{\mu a_3\ldots a_{10}]} + 9.4\tilde{G}^{(\alpha}_{\mu a_1a_2}C^{\beta\gamma)}_{a_3\ldots a_{10}]}\\
\tilde{G}^{\alpha}_{a_1\ldots a_{10}} &= \partial_\mu C^{\alpha}_{\mu a_1\ldots a_{10}}\,.
\end{aligned} \tag{17}$$

The field redefinitions of equation (16) contain all possible terms and the coefficients are fixed uniquely by requiring that the resulting Cartan forms can be expressed in terms of the field with p anti-symmetrised indices, the field with $p-2$ anti-symmetrised indices, $B^{(\alpha}_{[a_1a_2}$ and $\tilde{G}^{\alpha}_{\mu a_1a_2}$. That this can be done is non-trivial as there are fewer coefficients in the field redefinition of equation (16) than the number of terms required to be eliminated to bring the Cartan forms into the above form. The simplest way to see this is to change the coefficient of the third term in $\tilde{G}^{\alpha\beta}_{\mu a_1\ldots a_8}$ in equation (11) from -6.5 to an arbitrary number and then carry out the field redefinition to bring it to the required form; one finds that this is not possible unless the coefficient is -6.5. Similar restrictions hold for the ten form. Substituting the expressions of equation (17) into the field strengths of equation (15) we can compare the results with the field strengths of [11, equation (5.18)–(5.23)]. As we have just noted to bring the Cartan forms into the required form of equation (17) is already a non-trivial check. While some terms are not directly comparable due to possible field rescaling the ratio between the last two terms in $\tilde{G}^{\alpha}_{\mu a_1\dots a_6}$, $\tilde{G}^{\alpha\beta}_{\mu a_1\dots a_8}$ and $\tilde{G}^{\alpha\beta\gamma}_{\mu a_1\dots a_{10}}$ are independent of such transformations. We find that they are precisely those given by the E_{11} calculation carried out in this paper. The ratios associated with the six and eight forms were already contained in reference [10], but their uniqueness was not stressed.

It may seem that the ten form comparison with the two reference [11] is not legitimate as the field strength in the reference [11] has eleven indices and so each term vanishes identically. However, as explained in that paper the meaning of the field strength for these authors is that it invariant under the gauge transformation of the ten forms in any dimension, hence the unambiguous ratio is between the coefficients in the gauge transformation of the ten form in [11, equation (5.17)]. It is straightforward to verify that the ten form Cartan form $\tilde{G}^{\alpha\beta\gamma}_{\mu a_1\dots a_{10}}$ of equation (17) is invariant under the rigid transformation

$$\delta C^{\alpha\beta\gamma}_{a_1\dots a_{10}} = a^{\alpha\beta\gamma}_{a_1\dots a_{10}} - 9.4 C^{(\alpha}_{[a_1 a_2} a^{\beta\gamma)}_{a_3\dots a_{10}]} + 9 C^{(\alpha\beta}_{[a_1\dots a_8} a^{\gamma)}_{a_9 a_{10}]} + O(C^2)\,. \qquad (18)$$

One could have derived this transformation by carrying out an appropriate rigid g_0 transformation on the group element of equation (8) followed by the field redefinition of equation (16). As explained above we can convert this rigid transformation to a gauge transformation by taking $a^{\alpha_1\dots\alpha_r}_{a_1\dots a_p} = p\partial_{[a_1}\Lambda^{\alpha_1\dots\alpha_r}_{a_2\dots a_p]}$. Carrying out this last step and and then redefining the gauge parameter so as to bring it to the form given in reference [11] we find that

$$\delta C^{\alpha\beta\gamma}_{a_1\dots a_{10}} = \partial_{[a_1}\Lambda^{\alpha\beta\gamma}_{a_2\dots a_{10}]} - 2F^{(\alpha\beta}_{[a_1\dots a_9}\Lambda^{\gamma)}_{a_{10}]} + 8.4.3 F^{(\alpha}_{[a_1 a_2 a_3}\Lambda^{\beta\gamma)}_{a_4\dots a_{10}]} + O(C^2)\,. \qquad (19)$$

Comparing with [11, equation (5.17)] we find that the ratio in question between the last two terms is the same. Clearly, we could have carried out this comparison the other way round by converting the gauge transformation to the required rigid transformation. This throws light on the observation in reference [11] that the ten form field strength is invariant in any dimension, it is not so much to do with a symmetry that can be lifted in dimension, but more to do with the fact that the Cartan forms for the ten form, which are non-vanishing, are invariant under rigid E_{11} transformations.

The Cartan forms are inert under rigid E_{11} transformations, but transform under the local sub-algebra as $g^{-1}\partial_\mu g \to h^{-1}g^{-1}\partial_\mu g h + h^{-1}\partial_\mu h$. To form an object that transforms covariantly we introduce the operation $I(A) = I_c(-A)$ where I_c is the action of the Cartan involution. It acts on group elements as $I(k) = I_c(k^{-1}$ and $I(g_1 g_2) = I(g_1)I(g_2)$. The Chevalley generators behave under the Cartan involution as $I_c(E_a) = -F_a$ and $I_c(H_a) = -H_a$. Since the local sub-algebra is by definition invariant under the Cartan involution it follows that $I(h) = h^{-1}$. As a result, the quantity $U_\mu = g^{-1}\partial_\mu g + I(g^{-1}\partial_\mu g)$ transforms are $U_\mu \to h^{-1}U_\mu h$ while $w_\mu = \frac{1}{2}(g^{-1}\partial_\mu g - I(g^{-1}\partial_\mu g))$ behaves like a connection $w_\mu \to h^{-1}w_\mu h + h^{-1}\partial_\mu h$ The equations of motion are to be built from U_μ and w_μ so as to ensure their invariance under rigid transformations. As we will see below, we will be interested in first order equations, however, we note that $\partial_\mu U_\nu + [w_\mu, U_\nu]$ is second order in derivatives, but transforms covariantly. Looking at equation (12) we see that $U_\mu = S^1_\mu(E+F)+2S^2_\mu H_{10}+G^\alpha_{\mu a_1 a_2}(T^{a_1a_2}_\alpha - I_c(T^{a_1a_2}_\alpha))+\dots$ where $+\dots$ are higher level generators.

At the lowest level the local sub-algebra contains the Lorentz algebra and the U(1) sub-algebra of the SU(1,1) algebra. The latter U(1) has the generator $E_{10}-F_{10}$ and it transforms the Cartan forms as $\delta U_\mu = -a[E_{10}-F_{10}, U_\mu]$ where a is the local parameter. Introducing $S^\pm = S^1 \mp 2iS^2$ we find it transforms as $\delta S^{\pm\pm} = \pm 2iaS^{\pm\pm}$. The transformations of the other fields are most easily displayed by introducing the analogue to light-cone coordinates in the SU(1,1) index space;

$$\begin{aligned} T^{a_1a_2}_{\pm} &= \frac{1}{2}(T^{a_1a_2}_1 \mp iT^{a_1a_2}_2),\ T^{a_1\dots a_6}_{\pm} = \frac{1}{2}(T^{a_1\dots a_6}_1 \mp iT^{a_1\dots a_6}_2), \\ T^{a_1\dots a_8}_{\pm\pm} &= \frac{1}{4}(T^{a_1\dots a_8}_{11} - T^{a_1\dots a_8}_{22} \mp 2iT^{a_1\dots a_8}_{12}),\ T^{a_1\dots a_8}_{+-} = \frac{1}{4}(T^{a_1\dots a_8}_{11} + T^{a_1\dots a_8}_{22})\,. \end{aligned} \tag{20}$$

Their U(1) commutators are given by

$$\begin{aligned} [E_{10}-F_{10}, T^{a_1a_2}_{\pm}] &= \pm iT^{a_1a_2}_{\pm},\ [E_{10}-F_{10}, T^{a_1\dots a_6}_{\pm}] = \pm iT^{a_1\dots a_6}_{\pm}, \\ [E_{10}-F_{10}, T^{a_1\dots a_8}_{\pm\pm}] &= \pm 2iT^{a_1\dots a_8}_{\pm\pm},\ [E_{10}-F_{10}, T^{a_1\dots a_8}_{+-}] = 0\,. \end{aligned} \tag{21}$$

Introducing the analogous basis for the derivatives of the fields that appear in the Cartan forms

$$\begin{aligned} G^{\pm}_{\mu a_1a_2} &= \frac{1}{2}(G^1_{\mu a_1a_2} \mp iG^2_{\mu a_1a_2}),\ G^{\pm}_{\mu a_1\dots a_6} = \frac{1}{2}(G^1_{\mu a_1\dots a_6} \mp iG^2_{\mu a_1\dots a_6}), \\ G^{\pm\pm}_{\mu a_1\dots a_8} &= \frac{1}{4}(+G^{11}_{\mu a_1\dots a_8} - G^{22}_{\mu a_1\dots a_8} \mp 2iG^{12}_{\mu a_1\dots a_8}),\ G^{+-}_{\mu a_1\dots a_8} = \frac{1}{4}(G^{11}_{\mu a_1\dots a_8} + G^{22}_{\mu a_1\dots a_8}) \end{aligned} \tag{22}$$

and defining the U(1) charge by $\delta\bullet = [E_{10}-F_{10}, \bullet] - q\bullet$ where $\bullet$ is any of the above we find, using equation (20), that the expressions in equation (22) have the U(1) weights $\pm1, \pm1, \pm2$ and 0 respectively.

If we assume that the equations of motion for the gauge fields are first order in space-time derivatives they are then uniquely specified by demanding rigid E_{11} invariance, which is guaranteed by using the Cartan forms U, and invariance under the Lorentz and U(1) part of the local sub-algebra;

$$F^{\pm}_{a_1a_2a_3} = \frac{1}{7!}\epsilon_{a_1a_2a_3}{}^{b_1\dots b_7}F^{\pm}_{b_1\dots b_7},\ S^{\pm\pm}_a = \frac{1}{2.9!}\epsilon_a{}^{b_1\dots b_9}F^{\pm\pm}_{b_1\dots b_9},\ F^{+-}_{b_1\dots b_9} = 0\,. \tag{23}$$

These are the same equations are found in reference [12], except for the last equation, which constrains two of the three rank nine field strength to be equal. This last equation was given in reference [11]. It would be of interest to test the invariance of these equations at higher levels.

It was know [10] that the E_{11} non-linear realisation with only two of the three eight branes and all lower forms lead to the bosonic equations of motion of IIB supergravity. In this paper we have carried out the E_{11} non-linear realisation appropriate for the IIB theory including all the thee eight and ten forms and we have compared our results for the

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ten forms with those of reference [11] and found perfect agreement, including numerical coefficients. While the calculation given in this paper is just an exercise in E_{11} algebra, the results of reference [11] follow from the closure of the IIB supersymmetry algebra. There would seem to be no overlap between these two methods and so one can regard the results of this paper as a rather non-trivial check on the E_{11} conjecture.

We could have carried out the comparison with reference [11] in another way namely by simply computing the algebra of rigid E_{11} transformations converted these to gauge transformations and after a field redefinition carried out a comparison with the gauge transformation of reference [11]. However, the results will be the same as comparing the covariant objects as we have done in this paper.

The ten form does not possess a gauge invariant field strength so one might expect that it has trivial dynamics, nonetheless it does couple, in the supersymmetric Born-Infeld action, to a space-filling brane. This does have propagating field and as a result the ten form and its transformation properties do have consequences for the dynamics of the theory. In this context we note that it has been conjectured that the brane dynamics should also be E_{11} invariant [22].

In the table on page 27 of reference [12] the lowest level ten form is at the eighteenth (eleventh in terms of SU(1,1) multiplets) entry and has level (4,5) and so one has now confirmed the presence of fields in the adjoint representation of E_{11} which are relatively far down the table. It is also interesting to note that the E_{11} root associated with some of the ten forms has length squared -2 instead of the usual 2 that occur in finite dimensional semi-simple Lie algebras and the zeros that occur in affine algebra. A glance at the table shows that it also possess in the vicinity of the ten forms a SU(1,1) doublet of generators with the indices $R^{a_1\ldots a_9,b}$ and also a doublet of generators of the form $R^{a_1\ldots a_8,bc}$. It would be interesting to see if these can also be seen from the viewpoint of the IIB supersymmetry algebra. One could even wonder if one could find the dual gravity field in such a calculation.

As we have noted, at low levels the Borel sub-algebra generators in the decomposition of E_{11} to the IIB theory are in a one to one correspondence with the fields of IIB supergravity. As the latter can be assigned to either the NS-NS or R-R sector of the IIB string theory, we can assign the low level generators of E_{11} to either the NS-NS or R-R sector. It was observed in reference [13] that one can extend this classification to all the generators of E_{11} by taking the rule that the commutators admit a grading with the R-R generators being assigned as odd and NS-NS generators as even. Looking at the table on page 27 of reference [12] one see that a generator is even (odd), i.e. in the NS-NS (R-R) sector, if its associated root has an even (odd) number of α_{10}'s in its decomposition into simple roots. Put another way a generator with root α is in the R-R (NS-NS) sector if $\alpha.\Lambda_{10}$ is odd (even) where Λ_{10} is the fundamental root associated with node ten. As the roots add in any commutator this ensures the required graded structure. We note that $\alpha.\Lambda_{10}$ is just the level n_{10}. Given this rule it is easy to assign the ten forms to either the generalised NS-NS or R-R sector. Looking at the E_{11} decomposed to the $\hat{A}_9$ sub-algebra appropriate to the IIA theory given in the table on page 26 of reference [12] we find that a similar assignment is allowed and that the NS-NS sector has an even level corresponding to node ten and the R-R sector an odd level.

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The eleven dimensional, IIA and IIB theories all are non-linear realisations of E_{11}, but as there is only one E_{11} with a standard Chevalley presentation we can made a one to one correspondence between the three theories [13]. Looking at the table of reference [12] we see that all the ten forms in the IIB theory arise from the eleven dimensional theory at level four, which is one level above that for the dual graviton at level three and that below level three one only has the generators corresponding to the fields of eleven dimensional supergravity. In the IIB table we see that the ten forms have the E_{11} roots $(1, 2, 3, 4, 5, 6, 7, 8, 5, a, 4)$ with $a = 1, 2, 3, 4$. That for $a = 2$ has multiplicity two and these are easy to find in the IIA table of reference [12] as the two ten forms in that table at low level have a root of length squared -2, also with multiplicity two and precisely the same E_{11} root. That these ten forms are related by T-duality is known to the authors of reference [23]. For $a = 3$ which also has multiplicity two and length squared -2 we find the same root lower down the table, it corresponds to a IIA generator $\tilde{S}^{10}$ that is the highest $\tilde{A}_9$ states of $\tilde{S}^a, a = 1, \ldots, 10$. The $a = 4$ root also appears in the IIA table and we find it is the highest weight component of the generator $\tilde{R}^{(ab)}$. To find the last member of the IIB quadruplet we use the fact that E_{10} and F_{10} raise and lower respectively in the same SU(1,1) multiplet. In terms of IIA generators we have that $E_{10} = \tilde{R}^{10}$ where $\tilde{R}^a$ corresponds to the rank one gauge field in the IIA supergravity theory. Acting with $F_{10} = \tilde{R}_{10}$, the latter being the corresponding negative root, on the $a = 2$ generator we will find the $a = 1$ generator. This corresponds to the commutator $[\tilde{R}_{10}, \tilde{R}^{1\ldots10}]$ whose result is a generator with nine indices $\tilde{R}^{1\ldots9}$. However, this is not a highest $\tilde{A}_9$ representation and so will not occur in the table. The highest weight is $\tilde{R}^{2\ldots10}$ which is obtained by acting with $\tilde{K}^2{}_1 + \ldots + \tilde{K}^{10}{}_9$ which implies we must add the root $-(\alpha_1 + \ldots + \alpha_9)$. As a result, we find a nine form whose highest weight occurs in E_{11} as the root $(0, 1, 2, 3, 4, 5, 6, 7, 4, 1, 4)$, it is just the IIA nine forms $\tilde{R}^{a_1\ldots a_9}$ which is associated with the massive IIA theory. We note that the doublet of ten forms in IIB have the same roots as the $a = 2, 3$ members of the quadruplet and so correspond to one of the two ten forms and one of the two $\tilde{S}^{10}$'s in the IIA theory respectively.

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References

[1] E. Cremmer, B. Julia and J. Scherk, *Supergravity theory in 11 dimensions*, *Phys. Lett.* **B 76** (1978) 409.

[2] I.C.G. Campbell and P. West, $N = 2$ $D = 10$ *nonchiral supergravity and its spontaneous compactifications*, *Nucl. Phys.* **B 243** (1984) 112;
M. Huq, M. Namanzie, *Kaluza-Klein supergravity in ten dimensions*, *Class. and Quant. Grav.* **2** (1985) ;

F. Giani, M. Pernici, $N = 2$ *supergravity in ten dimensions*, *Phys. Rev.* **D 30** (1984) 325.

[3] J.H. Schwarz and P.C. West, *Symmetries and transformations of chiral* $N = 2$ $D = 10$ *supergravity*, *Phys. Lett.* **B 126** (1983) 301.

[4] J.H. Schwarz, *Covariant field equations of chiral* $N = 2$ $D = 10$ *supergravity*, *Nucl. Phys.* **B 226** (1983) 269;
P. Howe and P. West, *The complete* $N = 2$, $D = 10$ *supergravity*, *Nucl. Phys.* **B 238** (1984) 181.

[5] E. Cremmer and B. Julia, *The* $N = 8$ *supergravity theory, 1. The lagrangian*, *Phys. Lett.* **B 80** (1978) 48;
N. Marcus and J. Schwarz, *Three-dimensional supergravity theories*, *Nucl. Phys.* **B 228** (1983) 301;
B. Julia, *Group disintegrations*, in *Superspace and supergravity*, p. 331, S.W. Hawking and M. Roček eds., Cambridge University Press, Cambridge, 1981.

[6] H. Nicolai, *The integrability of* $N = 16$ *supergravity*, *Phys. Lett.* **B 194** (1987) 402; *On M-theory*, hep-th/9801090.

[7] B. Julia, in *Vertex operators in mathematics and physics*, Publications of the Mathematical Sciences Research Institute no 3, Springer Verlag, 1984; in *Superspace and supergravity* S.W. Hawking and M. Rocek eds., Cambridge University Press, Cambridge, 1981.

[8] P.C. West, *Hidden superconformal symmetry in M-theory*, *JHEP* **08** (2000) 007 [hep-th/0005270].

[9] P.C. West, E_{11} *and M-theory*, *Class. and Quant. Grav.* **18** (2001) 4443 [hep-th/0104081].

[10] I. Schnakenburg and P.C. West, *Kac-Moody symmetries of IIB supergravity*, *Phys. Lett.* **B 517** (2001) 421 [hep-th/0107181].

[11] E.A. Bergshoeff, M. de Roo, S.F. Kerstan and F. Riccioni, *IIB supergravity revisited*, *JHEP* **08** (2005) 098 [hep-th/0506013].

[12] A. Kleinschmidt, I. Schnakenburg and P. West, *Very-extended kac-moody algebras and their interpretation at low levels*, *Class. and Quant. Grav.* **21** (2004) 2493 [hep-th/0309198].

[13] P. West, *The IIA, IIB and eleven dimensional theories and their common* E_{11} *origin*, *Nucl. Phys.* **B 693** (2004) 76 [hep-th/0402140].

[14] T. Damour, M. Henneaux and H. Nicolai, E_{10} *and a 'small tension expansion' of M-theory*, *Phys. Rev. Lett.* **89** (2002) 221601 [hep-th/0207267].

[15] F. Englert and L. Houart, *G+++ invariant formulation of gravity and M-theories: exact BPS solutions*, *JHEP* **01** (2004) 002 [hep-th/0311255].

[16] M.R. Gaberdiel, D.I. Olive and P.C. West, *A class of lorentzian Kac-Moody algebras*, *Nucl. Phys.* **B 645** (2002) 403 [hep-th/0205068].

[17] P. West, *Very extended* E_8 *and* A_8 *at low levels, gravity and supergravity*, *Class. and Quant. Grav.* **20** (2003) 2393 [hep-th/0212291].

[18] P. Meessen and T. Ortin, *An* $\mathrm{SL}(2,\mathbb{Z})$ *multiplet of nine-dimensional type-II supergravity theories*, *Nucl. Phys.* **B 541** (1999) 195 [hep-th/9806120];
G. Dall'Agata, K. Lechner and M. Tonin, $D = 10$, $N = IIB$ *supergravity: Lorentz-invariant actions and duality*, *JHEP* **07** (1998) 017 [hep-th/9806140];
E. Bergshoeff, U. Gran and D. Roest, *Type IIB seven-brane solutions from nine-dimensional domain walls*, *Class. and Quant. Grav.* **19** (2002) 4207 [hep-th/0203202].

[19] A. Borisov and V. Ogievetsky, *Theory of dynamical affine and conformal symmetries as the theory of the gravitational field*, *Teor. Mat. Fiz.* 21 (1974) 329.

[20] A. Kleinschmidt and H. Nicolai, *IIB supergravity and E_{10}*, *Phys. Lett.* **B 606** (2005) 391, hep-th/0309198.

[21] J. Polchinski, *Phys. Rev. Lett.* **75** (1995) 184.

[22] P. West, *Brane dynamics, central charges and E_{11}*, hep-th/0412336.

[23] E. Bergshoeff, M. de Roo, B. Janssen and T. Ortin, *The super D9-brane and its truncations*, *Nucl. Phys.* **B 550** (1999) 289 [hep-th/9901055].

[24] F. Riccioni, *Spacetime-filling branes in ten and nine dimensions*, *Nucl. Phys.* **B 711** (2005) 231 [hep-th/0410185].

[25] Private communication from Fabio Riccioni to appear in a forthcoming paper by E. Bergshoeff, Mess de Roo, S. Kerstan, T. Ortin and F. Riccioni.